Investigación
Mathematical and Numerical Modeling in Maritime Geomechanics

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Abstract

A theoretical and numerical framework to model the foundation of marine offshore structures is presented. The theoretical model is composed by a system of partial differential equations describing coupling between seabed solid skeleton and pore fluids (water, air, oil,…) combined with a system of ordinary differential equations describing the specific constitutive relation of the seabed soil skeleton. Once the theoretical model is described, the finite element numerical procedure to achieve an approximate solution of the governing equations is outlined. In order to validate the proposed theoretical and numerical framework the seaward tilt mechanism induced by the action of breaking waves over a vertical breakwater is numerically reproduced. The results numerically attained are in agreement with the main conclusions drawn from the literature associated with this failure mechanism.

Keywords: Mathematical Modeling, Finite Element Method, Marine Structure foundation

1. Introduction

Engineers build various types of maritime structures: Breakwater and quay-walls for ports and harbors, seawalls and jetties for shore protection and platforms and rigs for the exploitation of oil beneath the seabed are some

examples.

The foundation design of these structures presents a series of difficulties due to the complexity of the cyclic forces exerted over the structure, derived from the dynamic swell action and transmitted to the seabed through a complex foundation-structure interaction, as well as the nonlinear soil behavior, where there is a coupling between solid skeleton and pore water.

Due to the complexity associated with the mechanism of seafloor dynamics it is essential to develop accurate and robust theoretical models to achieve realistic design solutions under a geotechnical engineering point of view.

As in many fields of science and engineering a theoretical model is a mathematical model that allows a representation of physical phenomena as accurate as possible. In geomechanics these mathematical models usually end up with systems of partial differential equations, systems of ordinary differential equations, systems of integro-differential equation, or combinations of them.

Closed-form solutions can often be difficult or even impossible to obtain for differential equations coming from engineering practice. Therefore it is indispensable to combine the theoretical models with numerical techniques in order to develop approximate solutions of the problem on hand.

In this paper we describe the procedure followed by applied mathematicians and geotechnical engineers [1-4] to develop robust engineering geotechnical designs of maritime offshore structures. Firstly, the principal theoretical components to be considered to properly reproduce the dynamics associated with a seafloor around and under a maritime structure are presented. This is accomplished in section 2. The procedure for a correct numerical approximation of the settled governing equations is presented in section 3. In section 4 a seaward tilt mechanism induced by the action of breaking waves over a vertical Breakwater is numerically reproduced. Finally some conclusions are presented.

2. Theoretical Modeling

2. 1 Introduction

There appear to be three major driving forces in the submarine environment of the continental shelf and slope area that may produce instability or movement in seafloor soils [5]:

- Gravity forces, i.e. influence of the sediment and offshore structures weight over seafloor.
- Hydraulic forces, i.e. influence of currents, tides, surface waves and internal waves over seafloor.
- Earthquakes and tectonic activity.

A theoretical model developed to reproduce accurately the influence of previously mentioned driving force over seafloor should contain the following fundamental components: i) a mathematical model to properly represent soil skeleton-pore fluids interaction, ii) an advanced constitutive model to reproduce the nonlinear soil behavior.

2. 2 Soil skeleton-pore fluid interaction mathematical modeling

Sea bed is usually modeled as a saturated poroelastoplastic media, composed by at least two constituents or phases, soil skeleton and pore fluid, each of them with an independent state of motion, leading to an interaction between them, i.e. a coupled system. In some cases sea bed pores might bear some occluded gas bubbles, raising the compressibility of the pore fluid. Among the different choices to describe this interaction behavior a macroscopic description of the phenomena is usually considered in geotechnical engineering modeling. This description rests over the volume fraction concept, i.e. porosity (Figure 1) where all geometric and physical quantities such as motion, deformation, and stress, are defined in the total control space, so they can be interpreted as the statistical average values of the real quantities. Therefore, the coupled domains are superimposed.

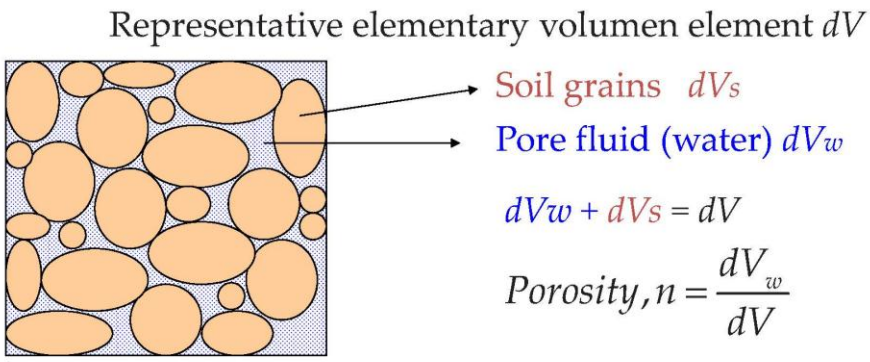

Figure 1. Description of the Porosity

Under this theoretical scope the following hypothesis are usually imposed, *i)* Lagrangian formulation for the skeleton (coordinate system moves with the solid phase) and Eulerian for the movement of the pore fluid relative to the skeleton (convective terms only appear for the relative movement of the fluid respect the skeleton), *ii)* saturated or slightly unsaturated soil (occluded gas bubbles are allowed to be within the pore fluid), *iii)* skeleton compressibility is much larger than the solid particles (usual assumption in soil mechanics). With these assumptions the fully dynamic Generalized Biot $w-u-p_w$ formulation for the soil skeleton-pore fluid interaction is obtained [6].

The $w-u-p_w$ formulation consists on a system of partial differential equations and includes the balance of linear momentum for the mixture (1), the balance of linear momentum of the pore water (2) and the mass conservation of the fluid flow (3).

$$\sigma_{ij,j} - \rho \cdot \ddot{u}_i - \rho_w \cdot \left[\ddot{w}_i + \dot{w}_i \cdot \dot{w}_{i,j} \right] + \rho \cdot b_i = 0 \tag{1}$$

$$-p_{w,i} - \frac{\rho_w \cdot g}{k_{ij}} \dot{w}_i - \rho_w \ddot{u}_i - \frac{\rho_w}{n^{sb}} \cdot \left[\ddot{w}_i + \dot{w}_i \cdot \dot{w}_{i,j} \right] + \rho_w b_i = 0 \tag{2}$$

$$w_{i,i} + \dot{\varepsilon}_{ii} + \frac{\dot{p}_w}{Q} = 0 \tag{3}$$

Where $\ddot{u}_i = \dfrac{\partial^2 u_i}{\partial t^2}$, etc., u_i is the soil skeleton displacement with $i = x, z$, for two dimensions and $i = x, y, z$ for three dimensions, σ_{ij} are the sea bed total Cauchy stress second order tensor components, p_w is the pore water pressure, $\dot{\varepsilon}_{ij} = \dfrac{1}{2} \cdot \left(\dot{u}_{i,j} + \dot{u}_{j,i} \right)$ sea bed rate of deformation tensor, $w_i = n \cdot \left(w_{fi} - u_i \right)$ is the average relative displacement of the fluid to the solid (w_{fi} is the actual pore fluid displacement), n sea bed porosity, $\rho = (1-n) \cdot \rho_s + n \cdot \rho_w$ combined density of the soil mixture, where ρ_s y ρ_w are the soil skeleton pore fluid densities, respectively, Q is the combine soil skeleton pore fluid compressibility, which is related with bulk modulus of each constituent through the expression $\left(1/Q\right) = \left(n/K_w\right) + \left((1-n)/K_s\right)$ with K_w the pore fluid bulk modulus and

K_s the sea bed bulk modulus, g is the gravity acceleration, k_{ij} Darcy permeability, while b_i are the volumetric forces per unit mass.

Balance of linear momentum equations are none other than the generalization of Newton's second law written locally for deformable materials while mass conservation express the mass variation inside the volume element as the transfer of mass in and out the element, with no diffusion nor production of mass [7].

In $w-u-p_w$ coupled equations flow and deformation are formulated including both the acceleration of soil skeleton and the acceleration of pore water relative to that of soil skeleton. If the acceleration of pore water relative to that of soil skeleton is neglected in the fully dynamic formulation, the $u-p_w$ formulation is obtained in which the soil skeleton displacement, u, and the pore water pressure, p_w, are the field variables. If both inertial terms, associated with the soil skeleton and the pore water are ignored, the Biot consolidation equation is attained.

The validity of these formulations has been studied by several researchers [8-11], concluding that each of them should be considered depending of the frequency of the driving forces, permeability and saturation degree of the seabed, and water deep. For instance, the quasi-static Biot formulation is considered as a good approximation to reproduce the effects associated with gravity forces while a $u-p_w$ formulation or even the fully dynamic $w-u-p_w$ might be essential to accurately reproduce the effects induced by wave motions and/or earthquakes.

2. 3 Constitutive modeling for seabed soils

It is well known that Newton's second law in particle mechanics cannot be solved until we know how the force depends on the position and velocity of the particle. Likewise, balance of linear momentum for the mixture in a continuum approach of porous media cannot be solved until we know how effective stress depends on the motion through a suitable strain expression. This missing relation is usually called the constitutive equation.

Under a mathematical point of view, a constitutive relation is defined by a set of ordinary differential equations. Methods for integrating them are usually classified as explicit or implicit. Implicit integration has been usually considered to exhibit significant advantages over explicit approaches as explicit integration of highly non-linear models may potentially lead to

inaccuracy and unstable behavior [12]. However, accuracy and efficiency might be enhanced by combining the explicit methods with automatic substepping and error control techniques [13, 14]. Moreover, explicit methods have shown some advantages compared with implicit strategies, i.e. no solution of a system of non-linear equations is required, only first derivatives are required in the formulation and usually are more straightforward to implement.

One of the most prominent aspects to achieve an accurate soil response is the choice of an appropriate constitutive model. Sea bed soil response under cyclic loading is the principal drawback concerning a constitutive relation of sea bed. The stress-strain law should be able to reproduce the soil degradation that takes the form of gradual resistance and stiffness changes with time, mainly due to repetitive loading. This degradation may cause sub-soil instability leading occasionally to structure collapse.

Classical plasticity theory based models like Von Mises, Druker-Prager, Cam-Clay, etc. are not able to reproduce plastic deformations induced by cyclic loading, due to the fact that after first load-unload cycle the subsequent ones (reloading-unloading) belong to the yield surface interior, i.e. elastic deformations take place, not being able to reproduce the possible soil degradation under repetitive loading.

Among the different possibilities to prevent this drawback we can mention a modified Cam-Clay model [15], plasticity models with isotropic-kinematic hardening [16], bounding surface models [17-19], bubble models [20, 21], Generalized Plasticity models [22, 23], etc. Among these the Generalized Plasticity present a high-quality simplicity and accuracy combination, being the theoretical framework considered for the stress-strain sea bed response in many researches [1-3].

2. 4 Boundary conditions

2.4.1 Introduction

The governing equation presented so far has to be complemented by suitable boundary and initial conditions. Boundary conditions should include the following ones:

- Hydraulic boundary conditions.
- Soil-Structure interaction conditions.
- Radiation boundary conditions.

2.4.2 Hydraulic boundary conditions

For these boundary conditions, distinction should be made between water-soil interface and water-structure interface.

If linear or non linear wave theory is considered to represent wave motion, water-soil interface boundary condition might be expressed explicitly through analytical expressions from potential flow theory, leading to a well defined boundary value problem. Instead, if breaking waves are to be considered, spectral or stochastic wave models might become necessary. Another possibility for a proper representation of breaking waves could be a numerical resolution of the Navier-Stokes equations by any of the existing advanced models, mostly based on VOF method [24], to simulate the interaction between wave trains and sea bed soil. Finally, if there are records available from wave gauges close to the area of interest, wave pressure might be estimated once a suitable wave theory is considered.

For the water-structure interface, apart from the mentioned techniques to deal with the water-soil interface there are expressions that permits the estimation of time-dependent pressures, forces and lever arms of the forces on the front faces and bottom of the maritime structure, both for breaking and non-breaking waves [25].

2.4.3 Soil-structure interaction modeling in maritime structures

Within the boundary conditions needed to complete the theoretical model, those concerning soil-structure interaction are essential to properly reproduce the principal loads transmitted to the foundation derived from gravity and hydraulic forces.

This contact interface has not been properly modeled in previous researches mostly represented through elastic mass-spring-dashpot models [26, 27], where the structure is considered as a point mass. Therefore, these models are not able to analyze different interface strain-stress states involved in the contact surface.

Other options considered in the past to represent this contact interface includes either prescribed loads by assuming complete flexibility of the structure or a prescribed displacements by assuming complete rigidity of the structure. These crude simplifications often lead to inaccurate predictions of the real behavior. Also, this soil–structure interaction might be modeled by joint elements. These elements typically use normal and tangential stiffness to model the pressure transfer and friction at the interface, defining a

constitutive relation within the joint element. Because they are predefined and their topology remains unchanged during the solution procedure, they are only suitable for predefined interfaces with small interfacial deformation.

While for linear and non linear wave induced soil response this interface might not have a paramount influence, this is not the case for impulsive actions derived from breaking waves where a highly variable complex interaction might be developed [2]. This highly variable complex interaction where large frictional sliding as well as surface separations and reclose might be involved, seems to be necessarily modeled through a frictional contact constrain model [28].

2.4.4 Radiation boundaries

When a dynamic analysis is performed in an unbounded region, as those associated with seafloor dynamics, artificial boundary conditions are needed to make the computational domain finite. The appropriate artificial boundary condition, radiation boundaries, for different wave problems is an important issue, since it must be designed to avoid the reflection in the finite computational domain of waves radiating towards the infinity.

In the field of the dynamics of saturated porous media, Gajo et al. [29] have developed a silent boundary extending the first and second order Higdom scheme to a saturated porous media under the $u-U$ Generalized Biot formulation [6]. Later on, a modification of the work done by Gajo et al. has been presented by Stickle [2], considering a first order Higdon scheme associated with the $u-p_w$ generalized Biot formulation.

3. Numerical modeling

Once the kinematic relations as well as the constitutive laws are integrated in the balance equations, a system of partial differential equations with associated field variables is established. Among the different numerical techniques to obtain approximate solutions of partial differential equations systems coming from engineering practice the Finite Element Method is one that has attained many achievements. The general procedures of the Finite Element discretization of equations are described in detail in various texts [9, 30, 31]. The principal characteristics of this technique are sketched in Figure 2.

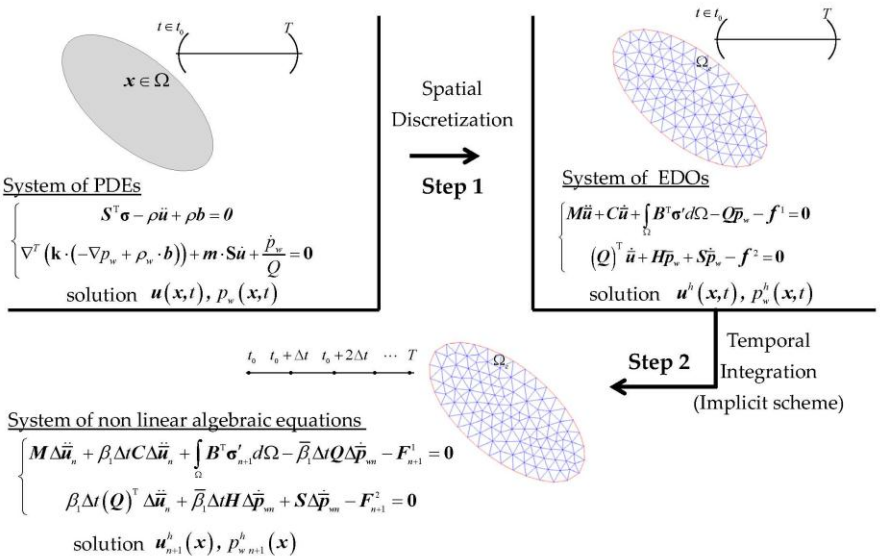

Figure 2. Description of the Finite Element procedure

Step 1. Spatial Finite Element discretization

The unknown functions are 'discretized' or approximated by a finite set of parameters, and shape function which are specified in spatial dimensions. Inserting the value of the approximating function into the differential equations we obtain a residual which is not identically equal to zero but for which we can write a set of weighted residual equations. A very suitable choice for the weighting function is to take them being the same as the mentioned shape function. Indeed this choice is optimal for accuracy in so called self-adjoint equations as shown in the basic texts and it is known as the Galerkin process. The proper choice of the element type in order to discretize the computational domain is of paramount importance. Under Babuska-Brezzi condition, mixed isoparametric elements should be considered with the appropriate number of nodes associated with each field variable.

Step 2. Temporal discretization

After spatial discretization through adequate interpolation functions, a second order ordinary differential equation system is obtained. The second order ordinary differential equation system needs to be discretized in time. Many time integration schemes are available in the specialized literature. Among these, the Generalized Newmark methods have been widely considered for the modeling of saturated geomaterials. Following this method, temporal discretization of the displacements involved (seabed and maritime structure skeleton) is performed by the Generalized Newmark

$GN22$ scheme while the excess pore pressure of the sea bed and possible permeable structures is discretized by the $GN11$ scheme [32], leading to a difference equation system. After incorporation this difference equation system in the second order ordinary differential equation a non linear algebraic system is obtained.

Step 3. Solution of a non linear algebraic system

Finally, the non linear algebraic system obtained needs to be solved in each time step through an iterative method like the Newton-Raphson scheme.

4. Vertical breakwater seaward tilt mechanism induced by breaking waves.

4.1. Introduction

In this section the seaward tilt mechanisms undergone by vertical breakwaters and induced by breaking waves is analyzed under the scope of the theoretical-numerical framework considered in the present chapter. This application has been mainly derived from the work done by Stickle et al. [2, 3].

Firstly a brief review of the conclusions drawn from the literature associated with the tilt mechanism is presented. Then the theoretical-numerical modelization is considered. Finally some results and discussions are established.

4.2. Seaward tilt mechanism

Vertical breakwaters are commonly used structures to protect harbors and sea shore from direct wave impact. The failure process of a vertical breakwater before the final collapse is often characterized by the progressive settlement and sea ward tilting. Experience obtained by many vertical breakwater failures have shown that seaward tilt is caused by inhomogeneous permanent settlement of the structure due to a cyclic asymmetric accumulation of permanent deformation of the subsoil beneath the breakwater. The deformation accumulation and strength degradation of the subsoil are mainly due to the cyclic reduction of effective stress associated with pore pressure build up.

Most seaward tilt mechanisms have been observed in actual breakwaters after the repetitive action of breaking waves generated within storms while

the subsoil is mostly fine loose sand. This combination of low wave period (breaking wave impacts), high characteristic drainage period $\left(T_{char,drain}\right)$ and low relative density are well known to be the natural setting for liquefaction or partial liquefaction in marine gravity structures [33]. Moreover, greater stress amplitude is observed under seaward than under shoreward caisson edge. This is due to triangular distribution of the uplift forces associated with breaking waves, with its maximum amplitude attained under seaward caisson edge [34]. This difference in load amplitude might induce an asymmetric permanent deformation of the subsoil beneath the breakwater.

4.3. Theoretical and Numerical modelization

4.3.1 Theoretical modelization

The soil-water-breakwater interaction has been modeled coupling three different physical systems with independent solution of each system being impossible without simultaneous solution of the others. These are caisson, rubble mound and sea bed (Figure 3).

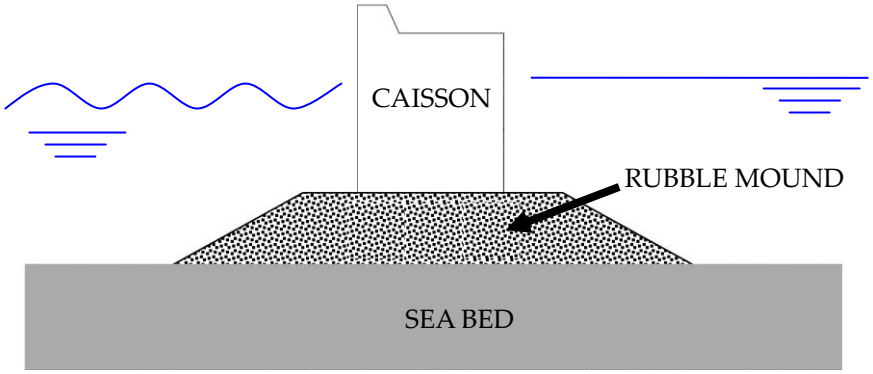

Figure 3. Physical systems involved in the soil-water-breakwater interaction model.

The mathematical model considered to represent skeleton-pore fluid interaction within the sea bed and the rubble mound is the Generalized Biot $u-p_w$ formulation, while the caisson has been considered as one phase media.

Regarding constitutive modeling, the seabed soil is considered as a *SandPZ* Generalized Plasticity media while the rubble mound and the caisson are considered to behave under a linear elastic law. Sea waves are not modeled as a proper physical system representing the sea wave actions exerted over the structure as boundary conditions. The theoretical model for

the soil-water-breakwater interaction proposed is developed in two dimensions under plain strain idealization.

The governing equation presented so far has to be complemented by suitable boundary and initial conditions. Figure 4 shows the contours where the boundary conditions need to be defined to complete the theoretical model for the soil-water-breakwater interaction proposed.

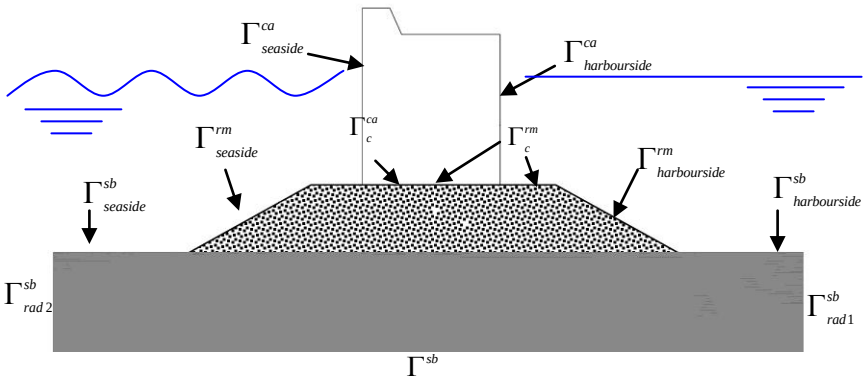

Figure 4. Localization of the contours to impose boundary conditions.

Boundaries $\Gamma^{sb}_{seaside}$, $\Gamma^{rm}_{seaside}$, $\Gamma^{ca}_{seaside}$, $\Gamma^{ca}_{harbourside}$, $\Gamma^{rm}_{harbourside}$, $\Gamma^{sb}_{harbourside}$

The direct contribution of the wave motion to the sea bed and rubble mound foundation has been neglected, only considering still water level pressure on the boundaries $\Gamma^{sb}_{seaside}$, $\Gamma^{rm}_{seaside}$, $\Gamma^{ca}_{harbourside}$, $\Gamma^{rm}_{harbourside}$, $\Gamma^{sb}_{harbourside}$. Only impact loading induced by breaking waves on the structure is considered.

Boundaries Γ^{sb}, Γ^{sb}_{rad1}, Γ^{sb}_{rad2}

An impermeable and rigid seabed bottom $\left(\Gamma^{sb}\right)$ is considered. This leads to a vanished fluctuation of all physical quantities. For the lateral boundaries $\left(\Gamma^{sb}_{rad1}, \Gamma^{sb}_{rad2}\right)$ a first order Higdon scheme associated with the $u-p_w$ generalized Biot formulation is considered. Regarding the pore pressure boundary conditions, the sea bed bottom Γ^{sb} and lateral boundaries Γ^{sb}_{rad1}, Γ^{sb}_{rad2} are considered impermeable.

Boundaries $\Gamma^{ca}_{seaside}$, Γ^{ca}_{c}

A Horizontal impulsive force due to breaking wave F_h and related uplift pressure F_u corresponding with regular waves defined by

$H = 0.6m,\ T = 6.5s,\ h_s = 1.6m,\ h_1 = 0.6m$ is applied. Time history impact loading corresponds to a typical single-peaked force associated with a very small or not air cushion wave breaking type, as shown in Figure 5. The action derived by ten breaking wave over the structure is considered.

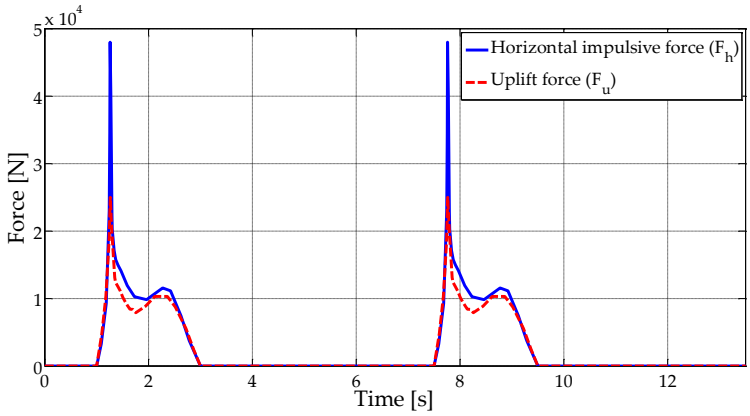

Figure 5. Time history impact loading shape considered for the numerical calculations.

The application point location of the horizontal impact force is considered usually constant and slightly under still water level, while uplift force applies at 1/4 of the caisson width from the seaward edge.

Boundaries Γ_c^{rm}, Γ_c^{ca}

Caisson-rubble mound contact interface has been modeled through a frictional contact constrain model limited to small relative sliding between contacting surfaces.

Initial Conditions

Regarding the initial conditions, still water level induced pore pressure is firstly established. Different stages associated with the rubble mound and caisson construction are performed through an elastoplastic consolidation process.

4.3.2 Numerical modelization

The geometry of the computational region including the spatial discretization mesh is shown in Figure 6. The mesh consists of 416 isoparametric triangular elements with 6 nodes quadratic interpolation for any skeleton displacement, $\boldsymbol{u}^{sb}$ (sea bed), $\boldsymbol{u}^{rm}$ (rubble mound) and $\boldsymbol{u}^{ca}$ (caisson), while 3 node linear interpolation for pore water pressure

interpolation in the sea bed and the rubble mound, p_w^{sb} , p_w^{rm} .

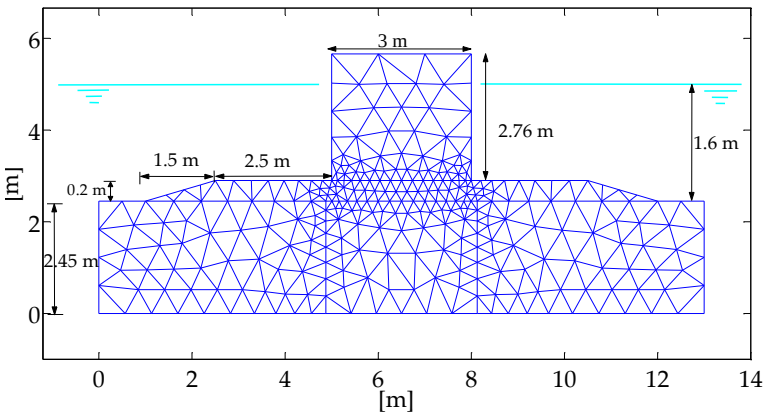

Figure 6. Geometry and mesh considered for the numerical calculations.

The boundary conditions considered for the numerical simulation are described in Figure 7.

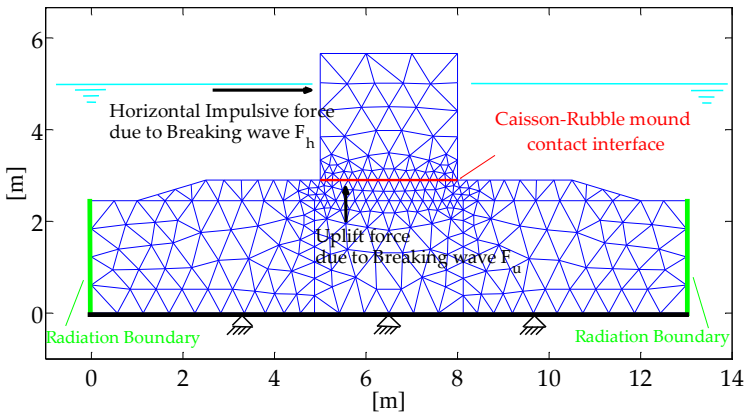

Figure 7. Boundary conditions considered in the numerical calculations.

All calculations are developed within MATLAB numerical environment.

4.4. Results and discussion

Different experimental results established a very close correlation between residual pore pressure and residual soil deformations beneath the breakwater due to caisson motion and induced by breaking wave impacts. In Figure 8 it is shown the relation between accumulated settlement (permanent

vertical displacement) and residual pore pressure numerically obtained.

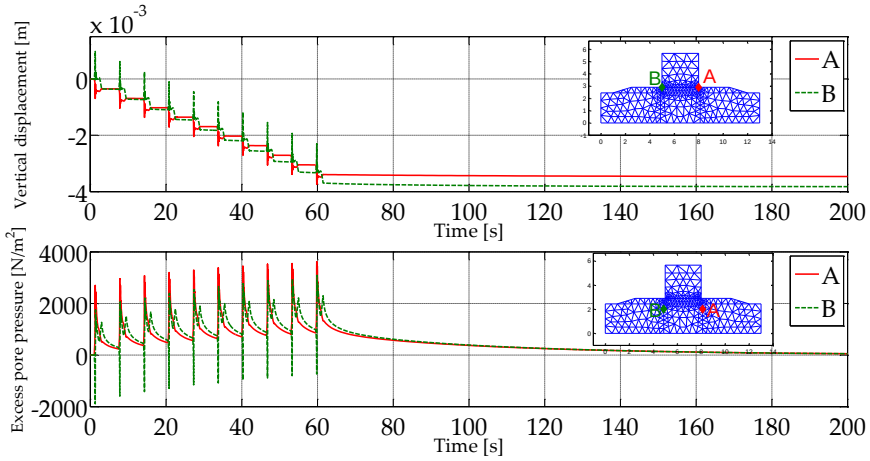

Figure 8. Relation between accumulated settlement and residual pore pressure (H=0.6m, T=6.5s, h_s=1.6m, h_1=0.6m). Numerical results.

The relation shown in Figure 8 indicates a residual pore pressure directly generated by the caisson motion induced by the impulsive wave action. The partial drainage occurring between two wave impact loads is not enough to dissipate the entire excess pore pressure generated, therefore a pore pressure accumulation process is developed. Just before the tenth impact load takes place, the accumulated excess pore pressure close to the sand layer surface is almost $0.8kN/m^2$. Once the impulsive wave action is finished, no extra excess pore pressure generation is performed but a pure dissipation process develops. While this dissipation process is taking place, the extra settlements observed induced by an elastoplastic consolidation process are negligible. After 200s the pore pressure derived by impulsive wave action dissipates completely in the vicinity of the sand layer surface.

Analyzing Figure 8, we observe a larger differential settlement at the seaward side than at the shoreward side. It is well known in geotechnical practice, when soils are loaded cyclically in the plastic range with nonzero mean stress they move towards the critical state line, describing cyclic accumulation of deformation. Experiment evidences show [35] that when a sample is loaded cyclically with constant mean stress, the greater the stress amplitude is the more mean stress decrease the sample accumulates. In the present case of a breakwater, the sand layer beneath the seaward edge is loaded with a greater stress amplitude than the one below the shoreward caisson edge, due to the uplift distribution, inducing a seaward settlement greater than the one observed at the shoreward.

In order to clarify the last aspect, Figure 9 shows the Von Mises equivalent

shear stress versus the mean effective stress at two different points A and B of the sand layer surface, under the shoreward edge (point A) and seaward edge (point B). The stress path direction observed under seaward and shoreward edges are almost opposite, while the shear stress amplitude is the double in point B than in point A. At the same time a clear mean effective stress reduction is observed at both locations, being slightly greater under the seaward edge.

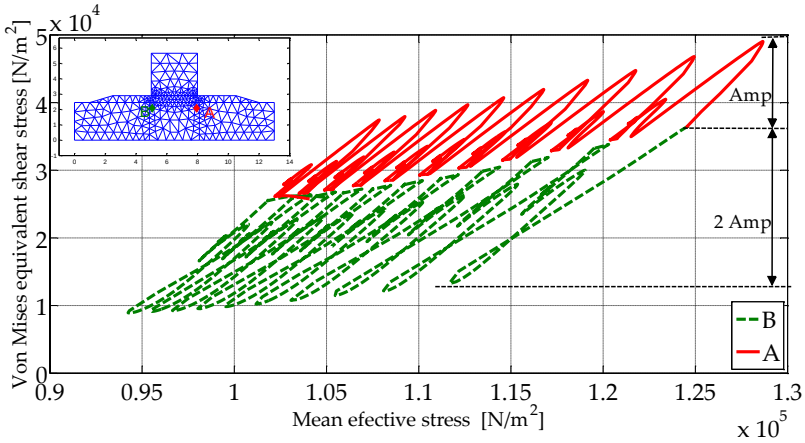

Figure 9. Von Mises equivalent shear stress versus the mean effective stress under the shoreward edge (point A) and seaward edge (point B).

The different stress amplitude observed under seaward and shoreward edges induced a more accentuated plastic behavior under the former as it is shown in Figure 10.

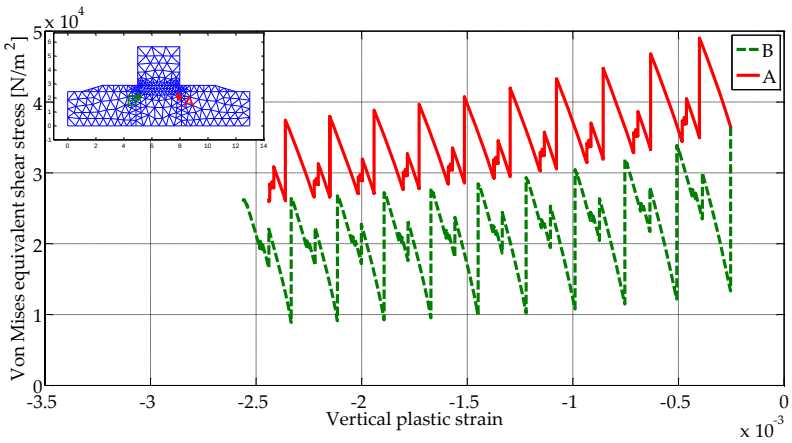

Figure 10. Von Mises equivalent shear stress versus vertical plastic strain under the shoreward edge (point A) and seaward edge (point B).

This asymmetric behavior leads to a greater permanent settlement in point B than in point A, i.e. seaward tilt mechanism, as it is shown in Figure 11

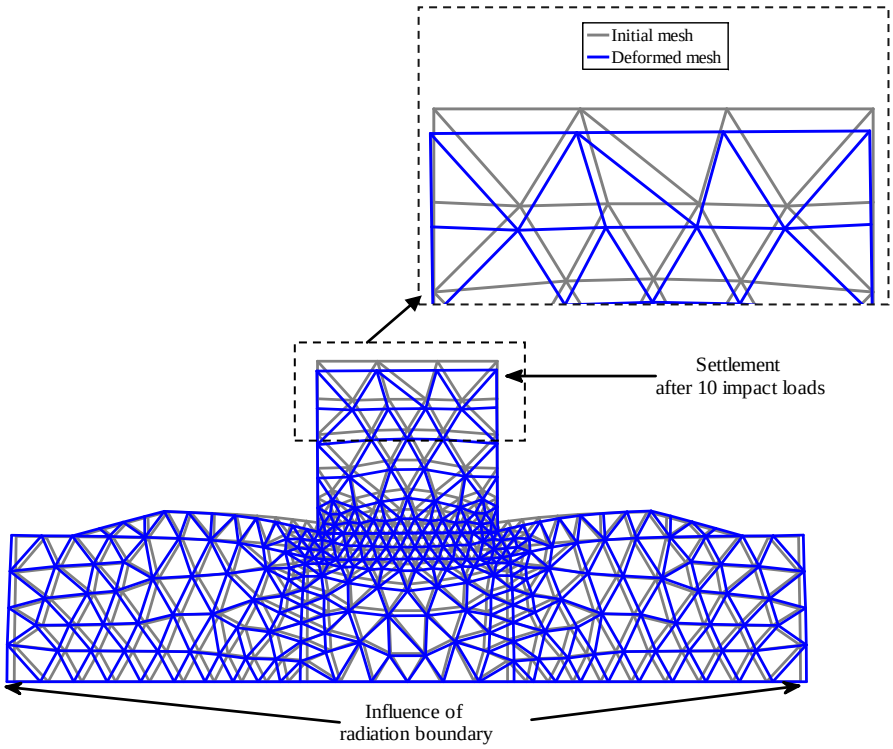

Figure 11. Seaward tilt induced by breaking waves

In this last figure the initial mesh (before the impulsive sea wave actions take place) and the deformed mesh (after the action of 10 breaking waves) are observed. It is clear that the vertical breakwater has suffered some settlement, being greater under the seaward edge of the caisson than under the shoreward part of the caisson.

5. Conclusions

In this paper the procedure followed by applied mathematicians and geotechnical engineers to develop robust engineering geotechnical designs of maritime offshore structures is described.

The principal theoretical components to be considered to properly reproduce the seafloor dynamics around and below a maritime structure are presented. An accurate maritime geotechnical modeling will drastically

depend on the consideration of these components.

Due to the complexity associated with the mechanism of seafloor dynamics it has been suggested the essential role play by numerical techniques in order to achieve realistic design solutions under a geotechnical engineering point of view.

Finally, one of the mechanisms that might eventually lead a vertical breakwater to failure, sea ward tilting, has been reproduced under the scope of the theoretical-numerical framework presented in this paper. The numerical results obtained are able to adequately represent the principal characteristics of this failure mechanism.

6. ACKNOWLEDGEMENTS

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