Experiencias Docentes

Application of Fractal Geometry in the construction of antennas: an assessment of activities in context by engineering students

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Abstract

This paper presents the students' perceptions about the implementation of a teaching proposal, based on the application of mathematics, in particular linear algebra applied to fractal antennas. The didactic interest in Fractal Geometry is recent, constituting a relevant problem, due to its enormous application possibilities. This geometry, very different from the Euclidean geometry, has not yet been widely addressed in Mathematics Education, although it is proposed in the curricula. We applied a 20-item survey to 26 students of a first year of engineering career, to gather their opinions. It was found that engineering students increased their motivation and took advantage of the potential of the proposal, by experiencing mathematics in engineering situations such as fractal antennas.

Keywords: mathematics in context, linear algebra, Fractal Geometry, multi-band antennas, engineering students.

1. Introduction

This work is part of a teaching and learning model that is being promoted by the Universidad Católica del Uruguay, which involves the development of transversal abilities in students through interdisciplinary activities. One of the thematic axes in the so-called Strategic Plan 2019-2021 is: "Excellence in interdisciplinary and transversal learning, facing a disruptive world" (p.3), within which the priority objective is to achieve transversal curricular integration of the different programs of the courses.

Traditionally, mathematics teachers in engineering programs tend~~s~~ to transmit knowledge with calculations that already exist in textbooks, solving problems with the students that know~~s~~ and repeat~~s~~ the shown method even if there is no creation on their part (Mendible, 2007). It should be noted that, in the training of future engineers, the study of formal mathematics is not an objective in itself, although they need to mathematize problems. This generates a cognitive conflict in the student who, in general, faced mathematics and engineering separately during his career (Camarena-Gallardo, 2009).

In this sense, it was proposed to teach some elements of Fractal Geometry (from now on FG) in two different subjects courses for university Engineers: in an introductory research project for the Industrial Engineering career, and in a linear algebra course for the second year of various Engineering careers (computer science, electronics, food, and audiovisual) at the Faculty of Engineering and Technology. In both cases, an activity was proposed that was developed with a different modality in each case, and consisted of the design, implementation and operation analysis of an antenna based on the fractal Sierpinski triangle. The design of the activity was carried out by professors from the Engineering Department and Exact and Natural Sciences Department.

In this work, the complete teaching proposal is presented, emphasizing those activities related to linear algebra, and the perception of students when addressing cross-curricular integration in a mathematics course is analyzed.

2. Framework: Math in context

This work focuses on the teaching-learning processes that occur when students face mathematics through its applications. This requires the development of an ability known as mathematical modelling, that is the creation or use of mathematical models when solving problems in context (Blum & Niss, 1991).

One of the educational objectives for the engineering students is to provide them with conceptual and functional tools that helps incorporating mathematical modeling as a cyclical process when solving an application problem (Mendible, 2007). An applied problem in mathematics is framed in a situation or context of the real world, as well as the questions that links mathematical concepts with said situation (Blum, et al., 1991). The implementation of this type of problems not only develops mathematical and modeling skills, but also generates greater interest in the subject and promotes diversified thinking in students (Alsina, 2007).

The theory called "Mathematics in the Sciences Context", is based on three main assumptions: mathematics is a tool in science and an educational subject, mathematics has a function specific to each level of education, and knowledge must be considered integrated (not fragmented) (Camarena-Gallardo P, 2009). Regarding the student's own activity, the objective is for them to be able to use mathematical knowledge in other areas that require it in their professional field.

One of the focuses of electronics has been miniaturization (Moore, 1965), and antennas have not been the exception. Effort is being devoted to making them small and to operate at different frequencies. Its structure must therefore include different sizes, and it must make efficient use of the space it occupies. It is pertinent to think that the antennas designed with FG can contemplate these characteristics: be multi-band (due to the self-similarity property) and very small (infinite lengths in finite areas).

FG was born in a context that gives great importance to geometry and visual mathematical analysis of real and concrete situations (measuring the shores of a country), in particular it began by studying aspects of nature. In other words, FG is naturally gifted for this approach to mathematics in the context of science.

2.1 Why Fractal Geometry in linear algebra?

In the field of mathematics teaching, FG is recognized for its potential to study and/or recover many mathematical notions, and for the great number of applications it has. However, an analysis of the researches that propose its teaching in high school and first years of university (Artigue, Fanaro & Lacués, 2021), showed that the way in which this geometry is taught is by making reference to the visual, and its aesthetic aspect. This way offers very few possibilities for a student to interact with these mathematical objects, without going further than calculating areas, perimeters, and fractal dimensions in very specific cases. Thus, for example, certain geometric shapes obtained in a fixed number of iterations are presented and introduced as a "fractal" without establishing that the fractal is the limit figure of that iteration.

FG studies geometric objects that are the product of iterating a procedure, either geometric or algebraic infinitely. There was (and still is) much controversy within mathematics itself about how to define these objects. The term fractal, from the Latin "fractus" (adjective meaning interrupted or irregular) was introduced by the mathematician Benoit B. Mandelbrot in 1975 (Dyson, 1978), who observed that nature is so complex that the Euclidean Geometry (EG) is not enough to study it, as in the case of natural forms such as a cloud, a mountain, or the coasts of countries.

In this way, FG allows us to describe a large part of the world around us, establishing mathematical models to study irregular and fragmented forms of nature, considering them as complex structures based on the repetition of simpler structures. There are several disciplinary fields that uses FG: medicine, geology, physics, technology, among others, and, in all of them, FG explains issues that EG does not reach (Fusi & Sgreccia, 2020).

In accordance with the strategic plan mentioned above and with the Syllabus of the linear algebra course taught in the different careers offered at the university, it is proposed to use the potential of FG to study fractal antennas.

An interesting starting point is to consider that fractals are figures, whose main characteristics are self-similarity and the intervention of two fundamental parameters that define the other key concept, that of fractal dimension: the number of parts into which the object is divided and the size of those parts (Castilblanco Hernández & Montana Páez, 2018). Thus, it is assumed that there are two elements that characterize the FG: self-similarity and dimension, in this case called fractal dimension.

The property of self-similarity is not fulfilled in the same way in any fractal, there are different types of self-similarity according to the number of points in which the presence of identical copies of itself can be appreciated (Artigue, et. al, 2021). But, if one is looking for a mathematical formulation of self-similarity that can be addressed in a linear algebra course, it is necessary to refer to the concept of similarity of EG to study those fractals that possess self-similarity of the strict type.

A similarity transformation in the plane is defined as a function of the plane in the plane obtained by composing a homothety with an isometry (rotation, translation or symmetry); these geometric transformations are studied in the linear algebra course from a matrix point of view and as linear transformations as well. For the study of fractals, these transformations must be contractive with a homothety ratio between zero and one, so that when applied they reduce the distance between any two points of the image figure.

These transformations must be applied iteratively, constituting a System of Iterated Functions (SIF). With the SIF mathematicians achieved a unity in such diversity, defining geometric transformations of the plane in the plane through affine transformations in a matrix form (Rubiano, 2009) using linear algebra resources.

A SIF must account for the transformations that are applied to the original figure called seed. It must provide the necessary information regarding the number of transformations that compose it and their characteristics, such as: the homothety ratio or contractivity ratio, the relative positions with respect to the seed, and their translation or rotation, the order in which they are applied.

Regardless of the original figure or seed, the limiting behavior of the SIF guarantees that each fractal algorithm gives rise to one limiting figure, and only one (Pérez Medina, 2007). Therefore, each set formed by similarity transformations defines a fractal image called SFI attractor, which always exists and is unique (Moreno-Marin, 2002). This aspect endows fractals with the property of strict self-similarity (Pérez Medina, 2007).

In the linear algebra course and in the classic texts of this subject, the concept of dimension is traditionally defined as the number of vectors that a basis of a certain Vector Space presents (Grossman & Flores-Godoy, 2019), that is, the extension of the concept of Euclidean dimension. In the case of the concept of dimension referring to fractal figures, an elementary construction for the self-similarity dimension, applicable to fractals with strict self-similarity, is the following: consider a straight line, a square and a cube (figures with Euclidean dimension one, two and three respectively). Each of the sides of the three figures is divided by a random number, take for example by four, i.e. in ratio $r=1/4$. Let N be the function that counts the number of congruent subsets as a function of $1/r$ when performing this division. For example, in the line $N(4)=4$ segments, or 4; in the square $N(4)=16$, or 4^2, and in the cube $N(4)=64$, or 4^3. The same construction can be performed if we change the ratio, in general, if $r=1/k$, we will obtain in each object, $N(k)= k^1$ segments, $N(k)= k^2$ squares and $N(k)= k^3$ cubes (Peitgen, Jürgens & Saupe, 2004) (Figure 1).

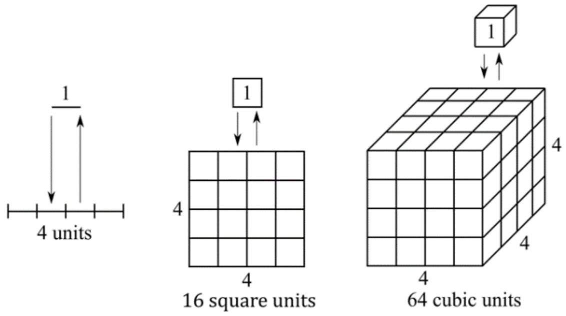

Figure 1. Self-similarity dimension with reason r=1/4. Source: Peitgen, et., al.

Considering the above, and from the knowledge we have of the Euclidean dimension, it can be established that the self-similarity dimension is the power d to which the similarity factor *k* must be raised. Starting from this idea, it is postulated that it is also valid to apply it for example, in the Koch Curve since it possesses strict self-similarity and therefore can be divided into congruent figures. Thus, if r=1/3, then N(3)=4, and therefore $4=3^d$ from which it follows that $d=log4/log3$ (Binimelis, 2017).

Although this reconstruction is the most used when teaching fractals, the proposal adopted a definition that does not require support in the EG and that involves the number of congruent parts into which each iteration of a fractal can be divided, and the similarity factor of each of these parts with the seed (Artigue, et. al, 2021).

3. The proposed activities and their implementation with linear algebra students

The linear algebra course developed in 2021 had a duration of one curricular semester (4 months), with distance modality due to the COVID-19 pandemic. It included a group laboratory practice to build a fractal antenna. The total number of students who participated was 26. Throughout the course, activities related to the FG that were proposed, are summarized in Table 1.

Table 1. Teaching activities

Name of the activity	Instruction	Goal
1. Videos about fractal antennas	Study a video about fractal antennas and identify their main advantages.	Introduce the mathematical concept of fractal and its characteristic properties.
2. Mathigon	Interact with the Mathigon website and carry out all the activities that are proposed in the book.	
3. Construction in GeoGebra	Design in GeoGebra the fourth iteration of the Cantor Set and Sierpinski Carpet fractals.	Identify the geometric transformations necessary for the construction of fractals. Determine these transformations in matrix form. Use the command ApplyMatrix in GeoGebra.

4. Self-similarity and Dimension	Use the definition proposed in the course notes of strict self-similarity and fractal dimension	Mathematically justify the strict self-similarity and the dimension of a fractal.
5. Design of a fractal antenna	Design in GeoGebra a fractal antenna based on the Sierpinski triangle given certain parameters.	Use geometric transformations, ApplyMatrix command or create new tool to design in GeoGebra a fractal antenna based on the Sierpinski triangle.
6. Construction of a fractal antenna	Build the antenna in a printed circuit on a copper plate.	
7. Study of the behavior of the built antenna	Use software-defined radios to analyze antenna behavior	Measure the characteristics of the designed antennas by using software defined radios (SDR) available in the Engineering Department.

The third activity was proposed after having dealt with matrix operations. Worked with matrix transformations, associating the geometric transformation and the corresponding matrix. The MatrixApply command in GeoGebra was used in some cases, in others the possibility of creating a "new tool". This provided the kick-off to determine the corresponding SIF for the Sierpinski triangle (Figure 2).

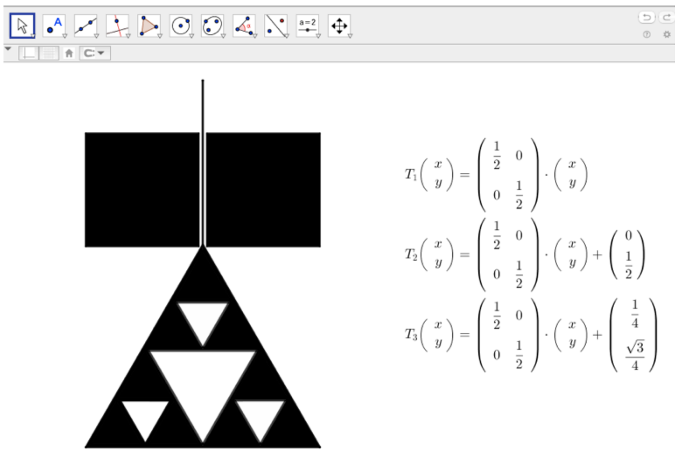

Figure 2. GeoGebra design of the antenna based on the Sierpinski triangle, production made by a student.

The sixth activity is based on the design and implementation of fractal antennas for UHF band, using the Sierpinski triangle (Sandoval & Vire, 2008). Although the size of the antenna is specified, it was modified and adjusted to the size of the copper substrate available in the electronics laboratory (10cm by 10cm), once the design was ready and the scales were verified, proceeded to the creation of the antenna. For this it was necessary to have the specific materials, which were: a plate, scissors, markers, copper board, acetate sheet and ferric acid. With these materials, the design was printed on the acetate sheet to pass through heat with iron to the copper plate (homemade sublimation process). Markers were used to re-mark the design in case the transfer had any details. Next the copper board was placed in a container with Ferric acid to remove the areas of copper that were not part of the antenna. The antenna connector was assembled with a numeric milling machine, in order to make the necessaries holes, then the connector was welded with tin to achieve electrical connection (Figure 3 shows the procedure performed by one of the groups).

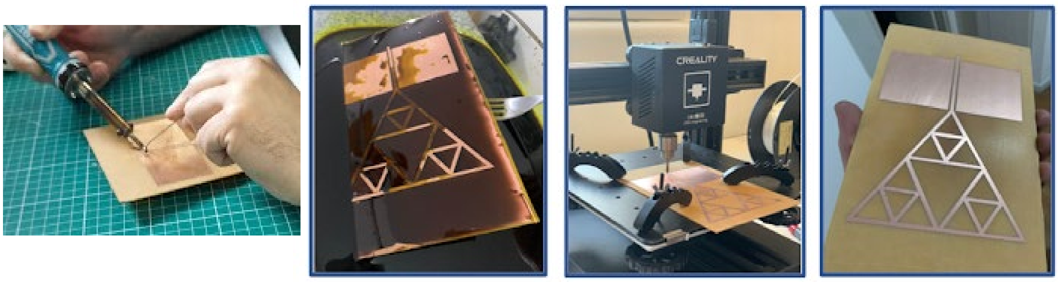

Figure 3. Antenna constructor procedure by students.

As last activities, the implemented antenna was measured using a HackRF software defined radio (Great Scott Gadgets) (Figure 4). The actual results were far from the expected most likely because of the rudimentary way the antennas were built. This is not an issue, this is not an Applied Electromagnetic or Antennas Design course, is just an excuse in the algebra course in order to achieve the predefined learning result or abilities.

Figure 4. Experimental setup using HackRF software.

4. Methodology

The objective of this work is to analyze the perception of the potentiality of the proposal to learn FG in a transversal way with the construction of antennas, by students of various engineering careers. To achieve this objective, the following question is formulated:

Is the mathematical-didactic potentiality of the proposal on FG perceived and developed by engineering students of linear algebra course?

Understanding the didactic-mathematical potentiality of the proposal as the possibility of:

- contributing to learning the concepts of self-similarity and dimension, as well as in the mathematical description of a fractal,
- positively valuing the use of technological tools such as applets, GeoGebra, videos and interactive sites among others,
- generate interest in the study of mathematics and, positively, appreciate the interaction of mathematics with situations specific to their training as engineers.

Within the evaluation of the linear algebra course corresponding to first-year engineering careers, a questionnaire was elaborated for the assessment of 26 students, who answered it at the end of the implementation of the proposal. At the beginning of the questionnaire, two questions were posed asking the students to evaluate the course, selecting one option (from bad to excellent) and justifying their selection. Then a set of statements were proposed for which each student had to rate with a Likert scale (Johns R., 2010) of four levels of agreement: Strongly Disagree (1), Disagree (2), Agree (3) and Strongly Agree (4). The questionnaire has paired items, one expressed in a positive sense and the other in a negative sense, asking about the same aspect.

5. Results

A descriptive and qualitative analysis of the responses to the general course evaluation questions, indicates that the course was evaluated as very good (50%) to excellent (50%). The assessment of the activities was for 85% of the students as interesting and very interesting. Among the aspects of the proposal that stood out the most, they mentioned collaborative work; the application of mathematics in fractal antennas; the use of software; the dynamism and laboratory practice; and fractals as something different in the mathematics class. Some student comments in the first course overview questions that exemplify these statements are as follows:

E1: "I found the topic of fractals very interesting, especially being able to apply it to concrete cases, and the same with matrices. Also, for abstract concepts like dimension, I feel that interactive activities like the Mathigon book helped a lot. It was also great to do the antenna, and it was a lot of fun when we dissolved an aluminum pan. I really enjoyed doing a project like this instead of a classic exam. I feel like I learned a lot and enjoyed the process, it wasn't a stressful closure like it usually is in other subjects."

E2: "I found the activities interesting and specifically the final assignment, as it takes us out of our comfort zone and participate in an activity that involves several areas and practical work which is entertaining."

E3: "I think the fractal topic was a good choice because it broke away from conventional math courses and thus was fun and more enjoyable."

Regarding the possibility presented by the proposal to contribute to the learning of the concepts of self-similarity and dimension and the description of a fractal, the results are encouraging. Although 69% of the students recognized some difficulty in constructing a fractal using elements of linear algebra (Q7), 77% of the students maintained that the definition of strict self-similarity presented facilitated their understanding of fractal dimension (Q4). In turn, 69% of the students stated that understanding that the fractal dimension can be a non-integer number is not so difficult (Q6). Finally, 81% of the students recognized the importance of their knowledge of basic geometry in explaining the construction of some fractals (Q3).

In the assessment of the use of the technological tools, almost all students (96%) indicated that the interactive materials helped in understanding the characteristics of fractals (Q1), although for slightly less than half of the students (46\%) the necessary knowledge of GeoGebra represented some difficulty (Q2). Despite this obstacle, 73% admitted that using GeoGebra was rewarding in constructing a particular iteration of a fractal (Q5).

As for the possibility of generating interest in the study of mathematics, 85% consider that studying fractals made them think about new issues, such as infinite processes, patterns or operations that repeat indefinitely (Q9). Almost all (92%) accept that fractals represent current mathematics with cutting edge technological applications (Q12), while 69% disagree with the idea that mathematics is too abstract and without applications (Q13).

Regarding the study of mathematics in context, almost all students (92%) positively valued the incorporation of fractal antennas in the course (Q16). More than half of them (62%) recognized that learning about fractals has a lot to do with their professional training (Q10). Thus 85% expressed that they would like to know more about fractals and their uses in other areas (Q11). Close to 73% agreed that studying mathematics in context represents an aid to their study (Q14). Almost 73% disagreed that the construction of the antenna was difficult even though it was done in a group (Q17), and 92% expressed that the collaborative work helped to make the construction of the antenna an easy task (Q18). Almost all students (96%) expressed excitement when they saw that the antenna worked (Q19), and the same percentage agreed with the importance of incorporating in-context activities in mathematics courses to strengthen skills for future engineers (Q20).

5.1 Quantitative analysis

Students Likert scores (R. Johns, 2010) per question item were determined. The internal precision of the sample was tested using Cronbach's alpha coefficient (Bartolucci F. et al., 2015), obtaining a value of $\propto = 0.827$. For each question, we formulated the following null hypothesis:

- H0: Engineering students of a linear algebra course did not perceive or develop the mathematical-didactic potentiality of the proposal on FG, since their mean has a score of 2.

- H1: Engineering students of a linear algebra course did perceive and develop the mathematical-didactic potentiality of the proposal on FG, since their mean has a score greater than 2.

A one-sided hypothesis contrast test was performed for each question, establishing a confidence level of 0.1% ($\propto=0.001$). The analysis was performed with the statistics software SPSS (IBM Corp) as presented in Table 2. The values presented show that all the hypotheses were rejected at a significance level of $p<0.001$. This allows us to conclude that the proposal was very well accepted for this population of engineering students, since scores of two or three (agreement and total agreement in the positive valuation statements) were obtained.

Table 2. Hypothesis Contrast Test Per Item.

One-Sample T Test

Test value = 2

	t	df	Significance P-value One-Tailed	Significance P-value Two-Tailed	Difference in means	99% Confidence Interval of the Differences Lower	99% Confidence Interval of the Differences Upper
Q1	14,423	25	<,001	<,001	1,615	1,30	1,93
Q2	3,638	25	<,001	,001	,577	,13	1,02
Q3	7,994	25	<,001	<,001	1,115	,73	1,50
Q4	6,374	25	<,001	<,001	1,000	,56	1,44
Q5	5,436	25	<,001	<,001	1,000	,49	1,51
Q6	3,942	25	<,001	<,001	,654	,19	1,12
Q7	2,004	25	,028	,056	,231	-,09	,55
Q8	6,429	25	<,001	<,001	1,038	,59	1,49
Q9	6,791	25	<,001	<,001	1,192	,70	1,68
Q10	3,682	25	<,001	,001	,615	,15	1,08
Q11	8,719	25	<,001	<,001	1,154	,78	1,52
Q12	10,916	25	<,001	<,001	1,346	1,00	1,69
Q13	5,204	25	<,001	<,001	1,000	,46	1,54
Q14	6,845	25	<,001	<,001	1,038	,62	1,46
Q15	5,354	25	<,001	<,001	,962	,46	1,46
Q16	10,718	25	<,001	<,001	1,269	,94	1,60
Q17	6,845	25	<,001	<,001	1,038	,62	1,46
Q18	12,127	25	<,001	<,001	1,538	1,18	1,89
Q19	12,559	25	<,001	<,001	1,423	1,11	1,74
Q20	14,423	25	<,001	<,001	1,615	1,30	1,93

Then, the Likert score per student was measured considering the 20 survey questions. Therefore, in this case the Likert scale was in the range of 20 (strongly disagree with all items) - 80 (strongly agree with all items), representing the distribution of the students' score according to the Likert scale, with a mean value of μ=61.42 and standard deviation of σ=7.34. Figure 4 shows the score histogram, where each bar contains the number of students who obtained the same score, where it can be noted that out of 26 students, 18 (70%) scored in the interval [54.076; 68.764] ([μ-σ, μ +σ]), and a total of 21 students (81%) between 54 and 73 points in total on the Likert scale, indicating that their assessment about the potentiality of the proposal was high.

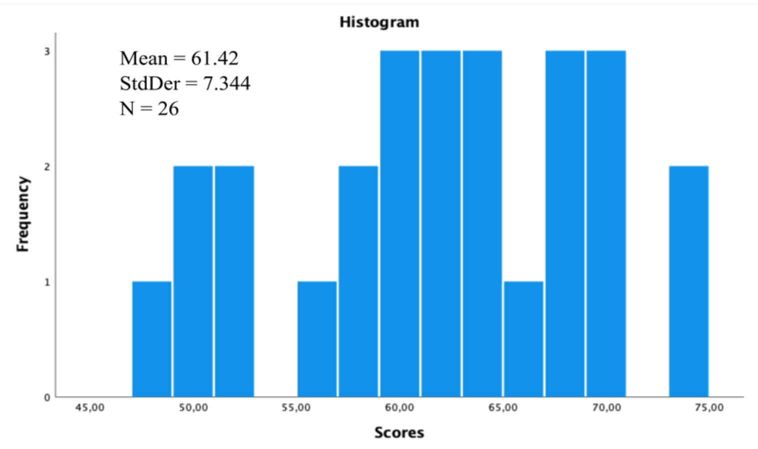

Figure 5. Distribution of the number of students according to the total score in the Likert Scale questionnaire.

6. Conclusions

The objective of this work was to know the assessment of engineering students of an interdisciplinary teaching proposal between mathematics and telecommunications. The focus of the proposal was the design, implementation and analysis of the operation of an antenna based on a known fractal object, involving professors from the Engineering Department and Natural Sciences Department. After implementing the proposal, students responded anonymously to a questionnaire that surveyed the extent to which they appreciated the didactic-mathematical potentiality of the proposal in their training as future engineers.

The analysis of the questionnaire indicates a very good acceptance of the teaching proposal on essential aspects of FG, since the students were enthusiastic about the activities with the software and the construction of the antenna. This offers an encouraging outlook regarding the possibilities that a multidisciplinary teaching will be well accepted by the students and will have a positive impact on their learning and their appreciation of Mathematics as a tool for modeling situations typical of their field of action as engineers.

Although these results encourage revising the proposal and making some changes involving the use of more sophisticated and specialized computer tools, such as the use of Phyton, which students at this level are familiar with, there are still some challenges for further research. For example, to design the greatest possible number of thematic units of the subject linear algebra, seeking to teach mathematics from a more genuine sense, which is to model engineering situations.

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